A New Fuzzy Sliding Mode Control Scheme

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Abstract **— In this paper, a new fuzzy sliding mode control scheme is proposed for second-order linear time-varying systems in order to obtain faster error convergence and desired error dynamics. It is well known that a second-order linear time-varying system can be used to model higher order linear time invariant systems piece-wisely. Therefore, it is necessary to do deep investigation for the stability and robustness analysis and the design of advanced tracking controllers for second-order linear time-varying systems. It is shown that the design of sliding mode control system is divided into two steps. A linear sliding mode controller is designed first to speed up the error convergence when the error is greater than one. A terminal sliding mode controller is then designed to guarantee that the error can converge to zero in a finite time when the error is around the system origin. In order to have a smooth switching from the linear sliding mode control to the terminal sliding mode control, a fuzzy logic technique is used to connect two sliding mode surfaces. The stability of the proposed fuzzy sliding mode controller is analyzed, and the convergence and robustness properties are demonstrated in a simulation example with a second-order system.**

Key Words**: Fuzzy control, sliding mode control.**

I. INTRODUCTION

In recent years, sliding mode control has been suggested as an approach for the control of systems with nonlinearities, uncertain dynamics and bounded input disturbances. The most distinguished feature of the sliding mode control technique is its ability to provide fast error convergence and strong robustness for control systems in the sense that the closed loop systems are completely insensitive to nonlinearities, uncertain dynamics, uncertain system parameters and bounded input disturbances in the sliding mode. It has been shown in [4] and [5] that the error convergence in linear sliding mode control systems is faster when the absolute values of errors are greater than one and then the error will asymptotically converge to zero when the time t tends to infinity. However the terminal sliding control systems have a different convergence property. For example, when the error is greater than one the error convergence is slower then the one in the linear sliding mode systems. After the error is less then one, the error can quickly converge to zero in a finite time in the terminal sliding mode control systems. Based on the above observation, we propose a new fuzzy sliding mode controller in this paper to combine the advantages of both linear sliding mode control and the terminal sliding mode control to further improve the error convergence. It will be shown that the design of sliding mode control system is divided into two steps. A linear sliding mode controller is designed first to speed up the error convergence. A terminal sliding mode controller is then designed to guarantee that the error can converge to zero in a finite time near the system origin. In order to have a smooth switching from the linear sliding mode control to the terminal sliding mode control, a fuzzy logic technique is used to connect two sliding mode surfaces. The stability of the proposed fuzzy sliding mode controller is analyzed, and the convergence and robustness properties are demonstrated in a simulation example with a second-order system.

II. PROBLEM FORMULATION

In this paper, we will design a fuzzy sliding mode scheme for the following second order SISO linear system.

$$\begin{cases} \dot{X} = AX + Bu \\ y = CX \end{cases} \quad (1)$$

where $X = [x_1 \quad x_2]^T$ is the state vector of the system, $B = [0 \quad 1]^T$, $C = [1 \quad 0]$ and

$A = \begin{bmatrix} 0 & 1 \\ a_1(t) & a_2(t) \end{bmatrix}$ and $|a_1(t)| < K_1$, $|a_2(t)| < K_2$, K_1, K_2 are positive constants.

The new control scheme is described as follow:

When the absolute value of e is bigger than e_b, linear sliding mode control is used to speed up the error convergence. When the $|e|$ is smaller than e_a, a terminal

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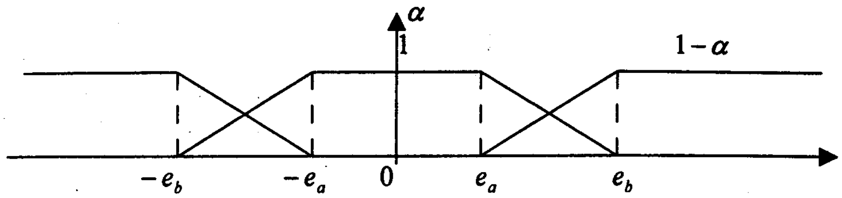


Fig. 1. Fuzzy Switching Function

sliding mode is employed to obtain a finite error convergence. A fuzzy switching function $\alpha(e)$ is introduced to smoothly change the sliding mode surfaces when the error e is between e_a and e_b or between $-e_a$ and $-e_b$.

For the linear sliding mode control part, the hype-plane variable is defined as $S_L = \dot{e} + he$. While for the terminal sliding mode control part, the hype-plane variable is defined as $S_T = \dot{e} + he^p$, where $p = p_1 / p_2$ and p_1, p_2 are positive odd integers, also $p_2 > p_1$, and h is a positive constant. The new fuzzy hype-plane variable is defined as:

$$\begin{aligned} S &= \alpha S_T + (1-\alpha)S_L \\ &= \alpha(\dot{e} + he^p) + (1-\alpha)(\dot{e} + he) \\ &= \dot{e} + h\alpha e^p + h(1-\alpha)e \end{aligned} \quad (2)$$

Remark 2.1: The fuzzy sliding mode variable S in (2) works according to the following rules: **(a)** When the absolute value of error e is smaller than e_a, $\alpha = 1$ and the fuzzy sliding mode variable in (2) is a pure terminal sliding mode variable, i.e. $S = S_T$. The error dynamics will converge to zero in a finite time. **(b)** When the absolute value of error e is bigger than e_b, the fuzzy sliding mode variable is a pure linear sliding mode variable, i.e., $S = S_L$. **(c)** When $e_a < e < e_b$,

$$S = S_3 = \frac{e_b - e}{e_b - e_a} \cdot S_T + \frac{e - e_a}{e_b - e_a} \cdot S_L$$

or when $-e_b < e < -e_a$,

$$S = S_4 = \frac{e + e_b}{e_b - e_a} \cdot S_T - \frac{e + e_a}{e_b - e_a} \cdot S_L$$

where $e = r - y = r - x_1$ is the error, and r is the reference signal and $\dot{e} = \dot{r} - \dot{x}_1 = \dot{r} - x_2$, $\ddot{e} = \ddot{r} - (a_1 x_1 + a_2 x_2 + u)$.

Similar to conventional sliding mode control technique, if the controller is designed such that S converges to zero, then we say that the switching plane variable S reaches the terminal sliding mode

$$S = \dot{e} + he^p = 0 \quad (3)$$

It has been shown in Zak[1] that $e = 0$ is the terminal attractor of the system (3). Let the initial value of e at $t = t_1$ (when the system states reach the switching plane) be $e(t_1)$, then the relaxation time t_i for a solution of system (3) is given as:

$$t_i = -\int_{e(t_1)}^{e \to 0} \frac{de}{he^p} = \frac{e(t_1)^{1-p}}{h(1-p)} \quad (4)$$

Since $1 - p = 1 - \frac{p_1}{p_2} = \frac{p_2 - p_1}{p_2}$, p_1, p_2 are positive odd integers, and $p_2 > p_1$, $(p_2 - p_1)$ is a positive even integer and $e(t_1)^{1-p} = \left[e(t_1)^{(p_2 - p_1)}\right]^{\frac{1}{p_2}} = |e(t_1)|^{1-p}$. Expression (4) can be written as:

$$t_i = -\int_{e(t_1)}^{e \to 0} \frac{de}{he^p} = \frac{|e(t_1)|^{1-p}}{h(1-p)} \quad (5)$$

Expression (5) also means that, on the terminal sliding mode in (3), the output tracking error converges to zero in a finite time.

III. CONTROLLER DESIGN

The design of the controller and the stability analysis using the proposed fuzzy sliding mode in (2) are stated in the theorem 3.1.

***Theorem 3.1*:** Consider the second order time-varying linear system in (1). If the controller is designed as:

$$u=\begin{cases} sign(S_T)\cdot\Delta_1 & -e_a<e<e_a \\ sign(S_L)\cdot\Delta_2 & e<-e_b, or, e>e_b \\ sign(S_3)\cdot\Delta_3 & e_a<e<e_b \\ sign(S_4)\cdot\Delta_4 & -e_b<e<-e_a \end{cases} \quad (6)$$

where:

$$\Delta_1 > \left|\ddot{r}-(a_1x_1+a_2x_2)+hpe^{p-1}\dot{e}\right| \quad (7)$$

$$\Delta_2 > \left|\ddot{r}+h\dot{r}-a_1x_1-(a_2+h)x_2\right| \quad (8)$$

$$\Delta_3 > \frac{1}{|e_b-e_a|}\cdot| h\dot{e}(e-e^p)+ \\ +(e_b-e_a)(\ddot{r}-a_1x_1-a_2x_2)+ \\ +h\dot{e}(pe^{p-1}(e_b-e)+(e-e_a))| \quad (9)$$

$$\Delta_4 > \frac{1}{|e_b-e_a|}\cdot| h\dot{e}(e-e^p)+ \\ +(e_b-e_a)(\ddot{r}-a_1x_1-a_2x_2)+ \\ h\dot{e}(pe^{p-1}(e+e_b)-(e+e_a))| \quad (10)$$

the system will then be stable and error will converge to zero in finite time.

Proof: Define the Lyapunov function as following:

$$V=\frac{1}{2}S^2 \quad (11)$$

When $|e|<e_a$: $S=S_T=\dot{e}+he^p$

$$\dot{V}=S_T\dot{S}_T=-S_Tu+S_T[\ddot{r}-(a_1x_1+a_2x_2)+hpe^{p-1}\dot{e}] \\ \le -|S_T|\cdot\Delta_1+|S_T|\cdot\left|\ddot{r}-(a_1x_1+a_2x_2)+hpe^{p-1}\dot{e}\right|<0 \quad (12)$$

When $e<-e_b$ or $e>e_b$: $S=S_L=\dot{e}+he$

$$\dot{V}=S_L\dot{S}_L=-S_Lu+S_L[\ddot{r}+h\dot{r}-a_1x_1-(a_2+h)x_2] \\ \le -|S_L|\cdot\Delta_2+|S_L|\cdot\left|\ddot{r}+h\dot{r}-a_1x_1-(a_2+h)x_2\right|<0 \quad (13)$$

When $e_a<e<e_b$:

$$S=S_3=\frac{e_b-e}{e_b-e_a}\cdot S_T+\frac{e-e_a}{e_b-e_a}\cdot S_L$$

$$\dot{V}=S_3\dot{S}_3=-uS_3+\frac{S_3}{e_b-e_a}\cdot[h\dot{e}(e-e^p)+ \\ +(e_b-e_a)(\ddot{r}-a_1x_1-a_2x_2)+ \\ +h\dot{e}(pe^{p-1}(e_b-e)+(e-e_a))]<0. \quad (14)$$

When $-e_b<e<-e_a$:

$$S=S_4=\frac{e+e_b}{e_b-e_a}\cdot S_T-\frac{e+e_a}{e_b-e_a}\cdot S_L$$

$$\dot{V}=S_4\dot{S}_4=-uS_4+\frac{S_4}{e_b-e_a}\cdot[h\dot{e}(e^p-e)+ \\ +(e_b-e_a)(\ddot{r}-a_1x_1-a_2x_2)+ \\ +h\dot{e}(pe^{p-1}(e+e_b)-(e+e_a))]<0. \quad (15)$$

Therefore, according to the Lyapunov stability theorem, the system is stable and the error will asymptotically converge to zero.

IV. A SIMULATION

A simulation of a second-order linear system is performed for the purpose of evaluating the performance of proposed control scheme.

$$\begin{cases} \dot{X}=AX+Bu \\ y=CX \end{cases}$$

where $X=\begin{bmatrix}x_1\\x_2\end{bmatrix}$, $A=\begin{bmatrix}0 & 1\\a_1 & a_2\end{bmatrix}$, $B=\begin{bmatrix}0\\1\end{bmatrix}$, $C=[1 \quad 0]$.

Choose

$$r(t)=1.25-\frac{7}{5}e^{-t}+\frac{7}{20}e^{-4t} \quad (16)$$

$$\dot{r}(t)=\frac{7}{5}e^{-t}-\frac{7}{5}e^{-4t} \quad (17)$$

$$\ddot{r}(t)=-\frac{7}{5}e^{-t}+\frac{28}{5}e^{-4t} \quad (18)$$

Let $p_1=3$, $p_2=5$, $p=p_1/p_2$=0.6, and

$$u = \begin{cases} sign(S_T)\cdot D_1 & -e_a < e < e_a \\ sign(S_L)\cdot D_2 & e < -e_b, or, e > e_b \\ sign(S_3)\cdot D_3 & e_a < e < e_b \\ sign(S_4)\cdot D_4 & -e_b < e < -e_a \end{cases} \qquad (19)$$

where

$$D_1 = \left|\ddot{r} - a_1x_1 - a_2x_2 + hpe^{p-1}(\dot{r} - x_2)\right| + 0.8 \qquad (20)$$

$$D_2 = \left|(\ddot{r} + h\dot{r}) - a_1x_1 - (a_2 + h)x_2\right| + 0.8 \qquad (21)$$

$$D_3 = \frac{1}{e_b - e_a}\cdot\left|\dot{e}(S_L - S_T) + (e_b - e)f_1 + (e - e_a)f_2\right| + 0.8 \qquad (22)$$

$$D_4 = \frac{1}{e_b - e_a}\cdot\left|\dot{e}(S_L - S_T) + (e + e_b)f_1 - (e + e_a)f_2\right| + 0.8 \qquad (23)$$

and $f_1 = \ddot{r} - a_1x_1 - a_2x_2 + hpe^{p-1}(\dot{r} - x_2)$,
$f_2 = \ddot{r} - a_1x_1 - (a_2 + h)x_2 + h\dot{r}$.

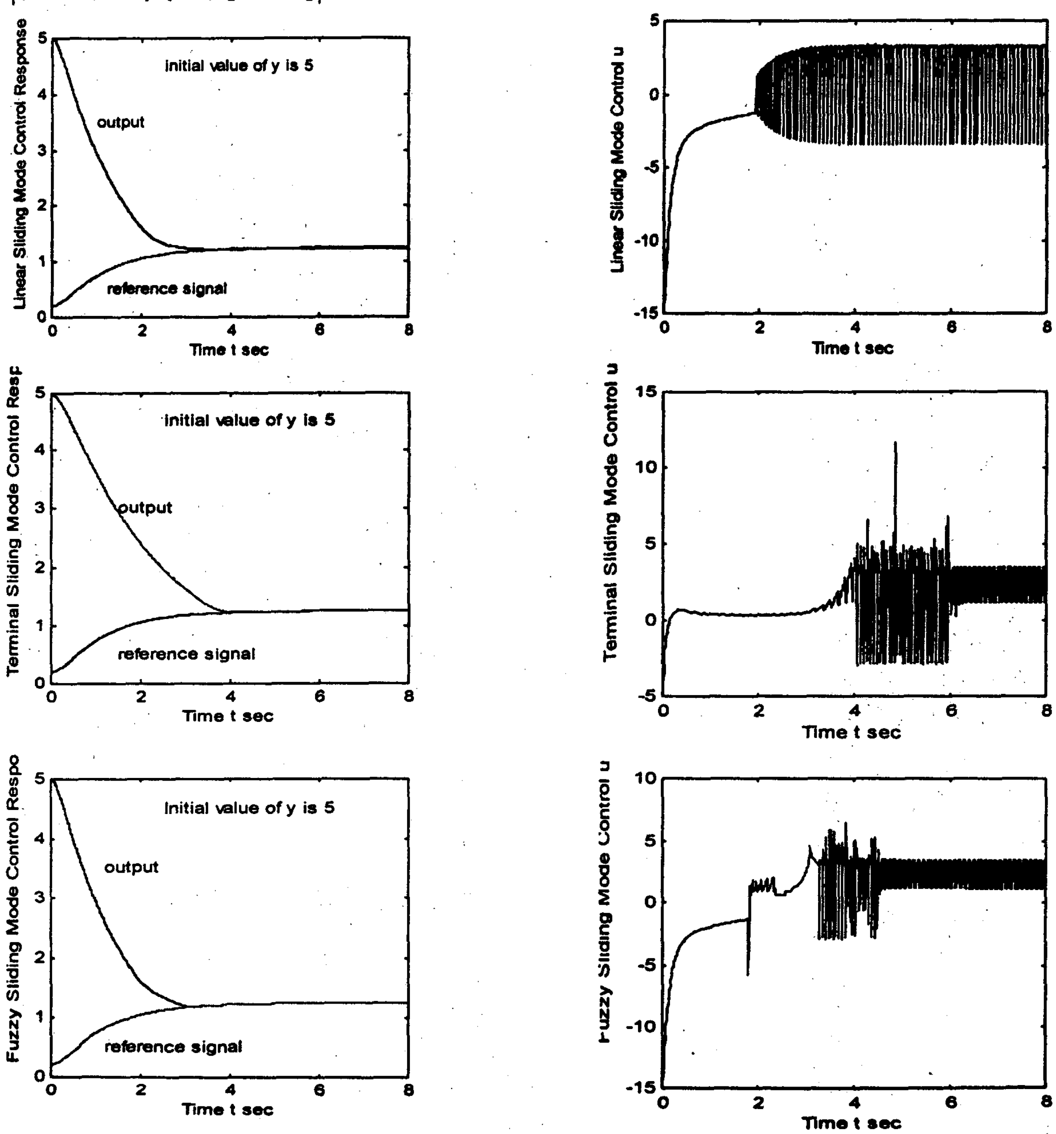
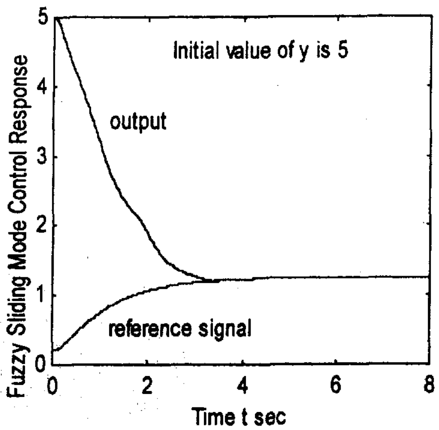


Fig.2. Simulation Results of second order system with constant parameters



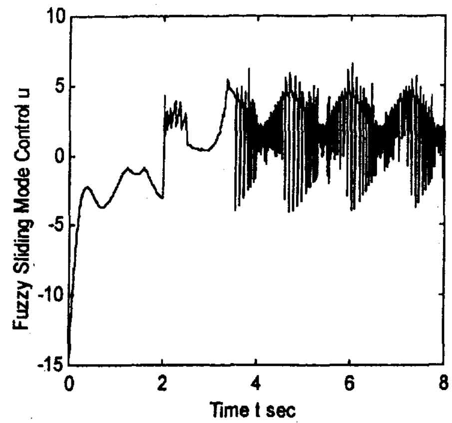


Fig.3. Simulation Results of second order system with time-varying parameters

The Runge-Kuta Method with the sampling interval ΔT=0.01sec is used. Fig.2 shows the simulation results with h=2, $a_1 = -2$, $a_2 = -5$, e_a=0.5, e_b=1. It is seen that the fuzzy sliding mode always has the fastest error convergence. Fig.3 shows the simulation results with h=2, $a_1 = -2$+sin(5t), $a_2 = -5$+cos(3t), e_a=0.5, e_b=1. It is seen that the fuzzy sliding mode always has the fastest error convergence though the values of a_1, a_2 are time varying.

V. CONCLUSION

A fuzzy sliding mode control scheme is proposed in this paper. The stability of the closed loop system has been proved and the fast error convergence has been shown in the simulation example.

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